# Segmentation of Edge Preserving Gradient Vector Flow: An Approach Toward Automatically Initializing and Splitting of Snakes

Chunming Li [1], Jundong Liu[2], and Martin D. Fox [1]

[1] Department of Electrical and Computer Engineering
University of Connecticut
Storrs, CT 06269, USA
{cmli,fox}@engr.uconn.edu

[2] School of Electrical Engineering and Computer Science
Ohio University
Athens, OH 45701, USA
liu@cs.ohiou.edu

## Abstract

*Active contours or snakes have been extensively utilized in handling image segmentation and classification problems. In traditional active contour models, snake initialization is performed manually by users, and topological changes, such as splitting of the snake, can not be automatically handled.*

*In this paper, we introduce a new method to solve the snake initialization and splitting problem, based on an area segmentation approach: the external force field is segmented first, and then the snake initialization and splitting can be automatically performed by using the segmented external force field. Such initialization and splitting produces multiple snakes, each of which is within the capture range associated to an object and will be evolved to the object boundary.*

*The external force used in this paper is a gradient vector flow with an edge-preserving property (EPGVF), which can prevent the snakes from passing over weak boundaries. To segment the external force field, we represent it with a graph, and a graph-theory approach can be taken to determine the membership of each pixel. Experimental results establish the effectiveness of the proposed approach.*

## 1. Introduction

*Active contours* [1], or *snakes*, are dynamic curves that move within an image domain to capture desired image features. There are two types of active contour models: parametric snakes and geometrical snakes. Parametric snakes are represented explicitly as parameterized contours and the snake evolution is carried out on the predetermined spline control points only. Geometrical snakes, on the other hand, are represented implicitly as the zero-level sets of higher-dimensional surfaces, and the updating is performed on the surface function within the entire image domain.

Since first introduced by Kass *et al.* [1] in 1987, numerous methods have been developed attempting to improve the performance of the parametric snakes, which includes to overcome the two major drawbacks: small capture range and lack of topological flexibility.

A number of methods have been proposed to address the capture range problem, including the distance potential forces [2], pressure forces [3], multi-resolution methods [4], and gradient vector flow (GVF) [5]. Among the above mentioned remedies, the GVF snakes [5] and GGVF snakes [6] proposed by Xu and Prince have gained tremendous popularity due to their ability of attracting the active contour toward object boundary from a sufficiently large distance and the ability of moving the contour into object cavities. However, these parametric snake models still cannot automatically handle topological changes.

To address the topological flexibility problem, Caseless *et al.* [14] and Malladi *et al.* [16] independently introduced geometric active contour models. These models are based on curve evolution theory [17,18] and level set method [19].

While the geometric snake models have gained great popularity, they have several inherent drawbacks when compared with parametric snake models. Firstly, the level-set scheme makes it difficult, if not impossible, to impose arbitrary geometric or topological constraints on the evolving contour indirectly through the higher dimensional hyper-surface [13]. Secondly, user-defined external force would be very inconvenient to be added. Thirdly, geometric active contour models may generate shapes that have inconsistent topology with respect to the actual object, when applied to noisy images with significant boundary gaps [7].

In light of the above inherent weaknesses of geometric active contour models, it is worthwhile to seek solu-

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tions within the parametric model realm. In [11], McInerney and Terzopoulos proposed a class of parametric deformable contours, called topology adaptive snakes or T-snakes based on an affine cell image decomposition (ACID) framework. The ACID framework provides a mechanism for contour reparameterization, which enables T-snakes to split or merge, adapting to the topology of target object. Recently, Ray *et al* [9] proposed a parametric active contour model that facilitates straightforward merging of multiple contours. They modify the GGVF vector field by adding a boundary condition, which is defined by the initial position of the evolving contours, to the GGVF PDEs, and use the solution as the external force field.

This paper is a continuation of efforts to overcome the above-mentioned difficulties of traditional parametric snake models in handling initialization and topological flexibility issues. Our approach is different from the existing methods in that we segment the external force field and then use the segmented enclosures for automatic snake initialization and splitting. First, an external force field is computed and segmented. Then, multiple snakes can be automatically initialized within each segmented enclosure. Similarly, one snake that encloses multiple objects can be automatically split into multiple snakes based on the segmented external force field. As a result of such automatic snake initialization or splitting, multiple snakes are produced and it is guaranteed that they will evolve towards the desired object boundaries.

In this paper, we propose an *edge preserving gradient vector flow (EPGVF)* as the external force field. Our EPGVF is an extension of the widely used GVF and GGVF fields. In addition to owning the desired properties of GVF and GGVF in granting snakes with larger capture ranges, the EPGVF model also possesses the unique advantage of preventing the snakes from passing over weak boundaries. The segmentation of the EPGVF field is implemented with a novel graph-theory approach. EPGVF is first quantized and then mapped onto a graph representation. Segmentation is thus reduced to finding the connected components in the graph, which can be efficiently implemented using a depth-first search.

## 2. Background

In the traditional parametric snake model [1], a snake is parameterized as $\mathcal{C}(x) = [x(s), y(s)]$, where $s \in [0,1]$ is the arc length parameter. The snake is led by certain forces moving through the spatial domain of an image to minimize the energy functional

$$E = \int_0^1 \frac{1}{2}(\alpha|\mathcal{C}'(s)|^2 + \beta|\mathcal{C}''(s)|^2)ds + \int_0^1 E_{\text{ext}}(\mathcal{C}(s))ds \tag{1}$$

where $\alpha$ and $\beta$ are pre-determined weighting parameters controlling the contour's smoothness and elasticity properties, respectively.

The $\mathcal{C}(s)$ that minimizes $E$ must satisfy the Euler-Lagrange equation:

$$\alpha\mathcal{C}''(s) - \beta\mathcal{C}''''(s) - \nabla E_{\text{ext}} = 0 \tag{2}$$

which can be solved by introducing a time variable to the contour $\mathcal{C}(s)$, and then finding steady solution of the following evolution equation:

$$\begin{cases} \frac{\partial}{\partial t}\mathcal{C}(s,t) = \alpha\mathcal{C}''(s,t) - \beta\mathcal{C}''''(s,t) - \nabla E_{\text{ext}}, \\ \mathcal{C}(s,0) = \mathcal{C}_0(s) \end{cases} \tag{3}$$

where $\mathcal{C}_0(s)$ is the initial contour. The term $\alpha\mathcal{C}''(s,t) - \beta\mathcal{C}''''(s,t)$ and the term $-\nabla E_{\text{ext}}$ in Eq. (3) are called internal force and external force, respectively.

The Euler-Lagrange equation in (2) can be written as

$$\mathbf{F}_{\text{int}} + \mathbf{F}_{\text{ext}} = 0 \tag{4}$$

where $\mathbf{F}_{\text{int}} = \alpha\mathcal{C}''(s) - \beta\mathcal{C}''''(s)$ and $\mathbf{F}_{\text{ext}} = -\nabla E_{\text{ext}}$. The internal force $\mathbf{F}_{\text{int}}$ controls the smoothness of the contour while the external force $\mathbf{F}_{\text{ext}}$ pulls the contour toward the object in the image.

As mentioned before, in the classical snake models, the external force field $\mathbf{F}_{\text{ext}}$ has very small capture range, which limits their utilities. To increase the capture range, Xu and Prince [5] introduced a new external force field to replace the traditional external force field $\mathbf{F}_{\text{ext}}$ as a gradient of a potential function. Their proposed external force field is called *gradient vector flow (GVF)*. The GVF vector field $\mathbf{v} = (u(x,y), v(x,y))$ is defined as the equilibrium solution of the following diffusion PDEs:

$$u_t = \mu\nabla^2 u - |\nabla f|^2(u - f_x) \tag{5}$$

and

$$v_t = \mu\nabla^2 v - |\nabla f|^2(v - f_y) \tag{6}$$

where $\nabla^2$ is the Laplacian operator, and $f(x,y)$ is the edge map. Typically, the edge map is defined as $f(x,y) = |\nabla G_\sigma * I(x,y)|^2$ for an input image $I(x,y)$, where $G_\sigma$ is the Gaussian kernel with standard deviation $\sigma$. In [5], Xu and Prince replaced the external force $-\nabla E_{\text{ext}}$ in Eq.3 with the gradient vector flow $\mathbf{v}$ for snake evolution. They also proposed an improved version of GVF, which is called GGVF [6].

While both GVF snake and GGVF snake [6] possess several desirable properties, such as significantly larger capture ranges and the ability of moving the contour into object cavities, they sometimes may fail to stop the snake at weak edges, especially at the location where a weak edge is very close to a strong one. Therefore, when a weak edge is located next to a stronger edge, the snake is very likely to pass the former and stop at the latter [8].


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## 3. Edge Preserving Gradient Vector Flow

In this section, we present an *edge preserving gradient vector flow* to overcome the above-mentioned drawback of GVF and GGVF. Let $I(x, y)$ be an image, and the vector $[I_x(x, y), I_y(x, y)]$ be the gradient vector of the smoothed image $G_\sigma * I(x, y)$, i.e., $[I_x(x, y), I_y(x, y)] = \nabla G_\sigma * I(x, y)$. The edge map $f(x, y)$ is defined as $f(x, y) \triangleq I_x^2 + I_y^2 = |\nabla G_\sigma * I(x, y)|^2$. It is easy to show the vector $\mathbf{p}$ given by

$$\mathbf{p} \triangleq [\frac{-I_y}{\sqrt{I_x^2 + I_y^2}}, \frac{I_x}{\sqrt{I_x^2 + I_y^2}}]^T \tag{7}$$

defines the edge orientation.

Now, we define the EPGVF as the vector field $\mathbf{v} = (u, v)$ that minimizes the following functional $\mathcal{E}$:

$$\mathcal{E}(\mathbf{v}) \triangleq \int\int [g(x, y)(|\nabla u|^2 + |\nabla v|^2) + h(x, y)(\mu |J_{\mathbf{v}}\mathbf{p}|^2 + |\mathbf{v} - \nabla f|^2)]dxdy \tag{8}$$

where $g(x, y)$ and $h(x, y)$ are the weighting functions; $\mu$ is a positive parameter, and $J_{\mathbf{v}}$ is the Jacobian matrix of the vector field $\mathbf{v}$. The first term $g(x, y)(|\nabla u|^2 + |\nabla v|^2)$ in Eq. (8) has an isotropic smoothing effect on the field $\mathbf{v}$, which is desired for homogeneous regions, but not for edge regions, as edges can be smeared out. This term is called the *smoothing term*. The second term $h(x, y)(\mu|J_{\mathbf{v}}\mathbf{p}|^2 + |\mathbf{v} - \nabla f|^2)$ consists of two components: $|\mathbf{v} - \nabla f|^2$ forces the field $\mathbf{v}$ close to the edge map gradient $\nabla f$; $\mu|J_{\mathbf{v}}\mathbf{p}|^2$ smoothes the field along the edge directions, but not across them. The entire term is thus called the *edge preserving term*. By choosing appropriate weighting functions $g$ and $h$, one can achieve the goal of extending the capture range and at the same time preserving object boundaries.

This advantage of the EPGVF over the GVF and the GGVF can be demonstrated by the experimental results shown in Fig. 1. In the input image shown in Fig. 1(a), there are two objects: an ellipse and a rectangle. The edge of the ellipse is weaker than that of the rectangle. In this example, we attempt to extract the boundary of the ellipse, and initialize the snakes as the same dashed contours, as shown in Figs. 1(c), 1(d), and 1(e). The snake evolution based on GVF, GGVF, and EPGVF is depicted in these figures, where the thick contours are the final snakes. As we can see, the EPGVF successfully stops the snake at the desired object boundary, while the GVF and the GGVF fail in doing this.

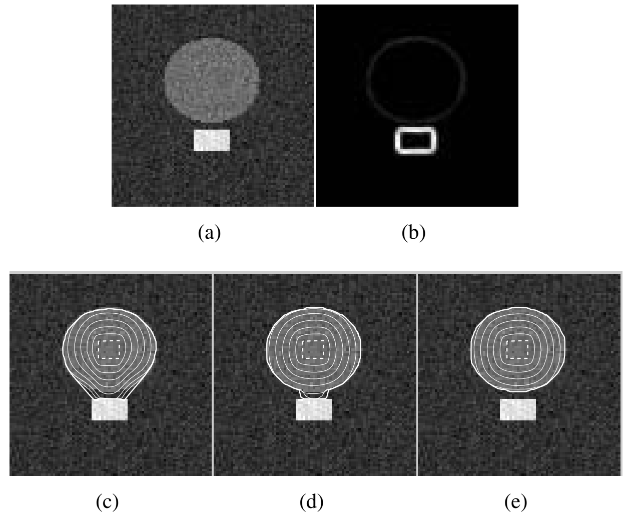


Figure 1. Performance of GVF, GGVF, and EPGVF. (a) The original image. (b) The edge map. (c) Evolution of the snake using GVF. (d) Evolution of the snake using GGVF. (e) Evolution of the snake using EPGVF

## 4. Segmentation of Edge Preserving Gradient Vector Flow

The goal of segmenting the external force field is to divide the image domain into disjoint regions. The snakes can thus be individually initialized within each of the enclosures and moved to the targeted object boundary within it, avoiding being misattracted by other objects. These enclosures define the *capture ranges* of the external force field.

A desired external force field should have a basic and important property: if a free particle is placed in this field, it should be able to move over a path and finally arrive at a wanted feature, e.g., a boundary or contour. To achieve the ultimate segmentation goal, we first construct an *Edge Preserving Gradient Vector Flow* field for the input image. By representing the force field with a graph, we can then partition the force field as well the image into different capture areas, by seeking for the maximal-connected components in the projected graph.

### 4.1. Quantization of Edge Preserving Gradient Vector Flow

For a given image, the feature map $f(x, y)$ and its associated external force field $\mathbf{v}(x, y)$ are defined on discrete variables $x$ and $y$. We denote by $\mathcal{D}$ the image domain $\mathcal{D} = \{(x, y)|x = 1, 2, \cdots, M, y = 1, 2, \cdots, N\}$. After the EPGVF is computed, the rest of work can be done in two steps: 1) map the vector field into a graph and 2) the seg-

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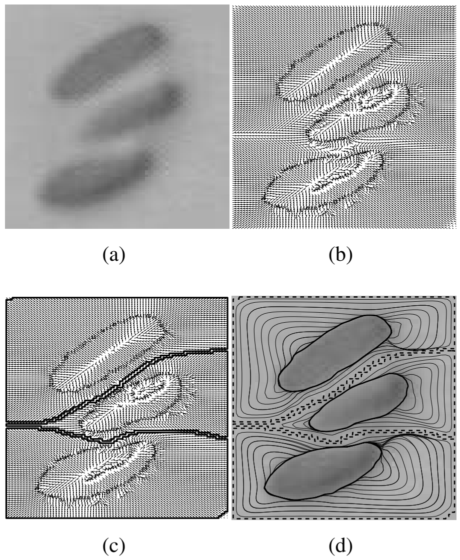

Figure 2. Results of a microscope image of pollen. (a) Original image. (b) The EPGVF field. (c) The segmented EPGVF vector field. (d) Initialization and evolution of multiple snakes

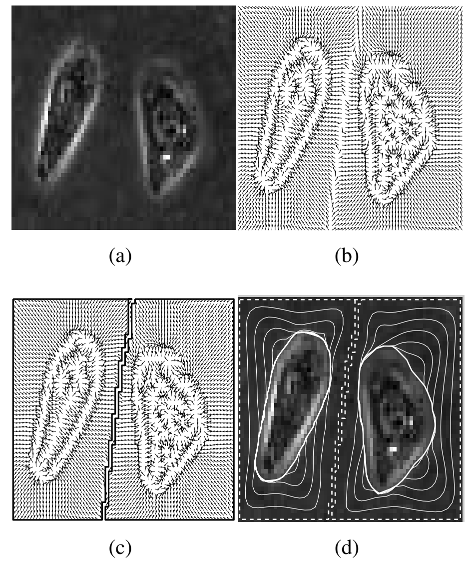

Figure 3. Results of a microscope image of cells. (a) Original cell image. (b) The EPGVF field. (c) The segmented EPGVF vector field. (d) Initialization and evolution of multiple snakes.

mentation of EPGVF is later obtained by finding the connected components in the graph. In implementing step 1, we quantize the normalized EPGVF $\mathbf{v}$ as follows: for each point $P = (x, y)$ in $\mathcal{D}$, we choose one of the eight neighboring points, say $Q$, that makes vector $Q - P$ closest to the direction of $\mathbf{v}(P)$. Then we define

$$\tilde{\mathbf{v}}(P) \triangleq Q - P \tag{9}$$

a quantized vector field in the image domain $\mathcal{D}$. This vector field $\tilde{\mathbf{v}}(x, y)$ gives the direction in which a free particle can move from a point $P = (x, y)$ to the adjacent point $P + \tilde{\mathbf{v}}(P)$.

### 4.2. Graph Representation and Segmentation of External Force Field

Given a feature map $f(x, y)$ and the corresponding quantized external force field $\tilde{\mathbf{v}}(x, y)$, we set a fixed threshold value $T$. The directed graph $(V, E)$ is then constructed, as $V$ contains all the image points in domain $\mathcal{D}$ and $E$ is the set of all the ordered pairs $(P, Q)$ that satisfy one of the following conditions:

**1.** $Q = P + \tilde{\mathbf{v}}(P)$.

**2.** $f(P) > T$ and $f(Q) > T$, and $P$ and $Q$ are adjacent points in $\mathcal{D}$.

With the graph $(V, E)$ constructed in this way, the segmenting of the external force field is reduced to the problem of finding weakly connected components in the graph. The depth-first search algorithm [10] can be used to achieve this end.

## 5. Applications of Segmented Edge Preserving Gradient Vector Flow

In this section, we demonstrate the improvement made on automatic initialization of multiple snakes, as well as on automatic snake splitting.

### 5.1. Automatic Initialization of Multiple Snakes

The segmented external force field can be used to initialize snakes automatically. For an input image with $N$ objects, the segmentation of the derived external force field results in $N$ disjoint capture ranges $\mathcal{R}_i$, $i = 1, \cdots, N$. For each capture range $\mathcal{R}_i$, a snake can be automatically initialized as the boundary of $\mathcal{R}_i$, denoted by $\partial\mathcal{R}_i$. Each of these initial snake is within the capture range $\mathcal{R}_i$, and will be evolved to the associated object boundary. Fig. 2 shows the result for a microscope image of pollen. The original image is shown in Figs. 2(a). The EPGVF vector field is shown in Fig. 2(b). The segmentation of the EPGVF field results in three capture ranges, as shown in Fig. 2(c), where the three dark contours are the boundaries of the capture ranges. Three snakes are automatically initialized as the boundaries of these capture ranges. The evolution of these snakes is shown in Fig. 2(d), where the dashed contours are the automatically initialized snakes and the thick solid contours are the final snakes. Another example using a microscope image of cells is depicted in Fig. 3. As we can see, all the object boundaries, including the weak edges, are successfully captured in the above experiments.

### 5.2. Automatically Splitting Snakes

Our segmented EPGVF can also provide a straightforward approach for snake splitting, which usually cannot be gracefully handled by traditional parametric snakes. If a single snake encloses $N$ objects of interest, it has to be split into $N$ smaller snakes in order to capture all these objects. Let $\Omega$ be the region enclosed by the snake. Then we can use the boundaries of the intersection of $\Omega$ and each of the capture ranges, namely $\partial(\Omega \cap \mathcal{R}_i)$, as the new snakes. Thus, the original snake is split into $N$ snakes. Since each of these snakes is within one capture range $\mathcal{R}_i$, it will finally evolve to the corresponding object boundary.

The snake splitting can be performed anytime during the evolution of the snake, such as the moment when it converges to a certain configuration and cannot further progress toward the object boundaries. This process can be demonstrated by the experimental results shown in Fig. 4.

## 6. Conclusions

In this work, we have developed a new approach to overcome the limitation of traditional parametric snakes in initialization and topological changes. In our approach, the external force field is segmented first, and then the segmented enclosures are used for automatic initialization and splitting of the snake. Such initialization or splitting produces multiple snakes, each of which is within the capture range associated to an object in the image and will be evolved to the object boundary. The segmentation of the external force field can be implemented effectively by a graph-theory approach.

This paper also presents an edge preserving gradient vector flow (EPGVF) as external force field for snakes. The EPGVF overcomes the drawback of boundary smearing associated with GVF and GGVF. Moreover, the segmented EPGVF field provides a novel and effective mechanism for automatic initialization and splitting of snakes. Our experiments using real and synthetic images have produced promising results.

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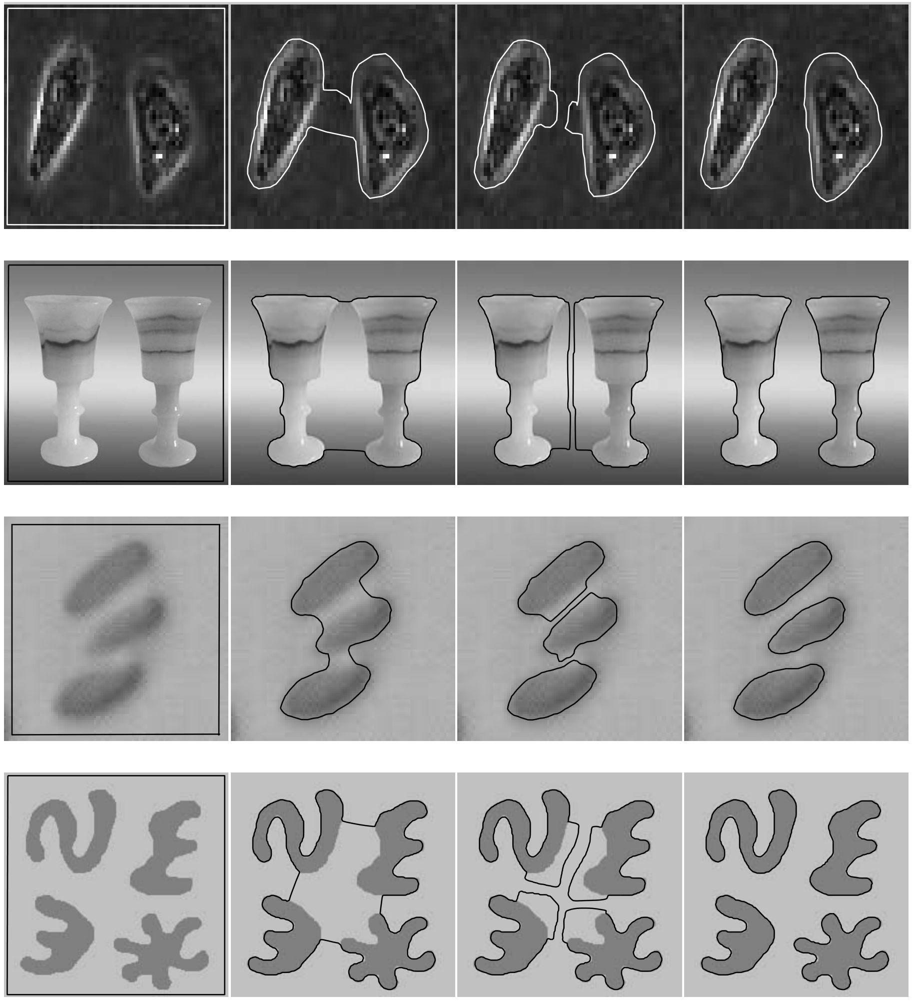

Figure 4. Splitting snakes by using the segmented external force field. The first column: the original images and the initial snakes. The second column: the evolving snakes before splitting. The third column: the snakes after splitting. The fourth column: the results of the evolution of multiple snakes.


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